interesting to compare the rainfall distribution map (fig. 9) of this storm with the maps of the four typhoons just discussed. Prof. A. J. Henry [6] has pointed out in connection with this storm that the distribution of rainfall in a tropical storm is different from that in an extratropical storm. In the latter the rain is not distributed so uniformly about the center as in the former. This storm, he continued, maintained its tropical character until it reached the upper Ohio Valley. The heavy rainfall in Galveston and Houston should be noted (see Table 2), as these stations are on the coast where the storm passed from sea to land. In this connection it might be mentioned that during the New Orleans hurricane of September 29, 1915, New Orleans had a precipitation of about 209 mm. (8.26 inches) within 24 hours on September 29.

TABLE 2.—*Amount of rainfall received during the Galveston hurricane of Aug. 13–23, 1915.*

Place.	Amount.		Place.	Amount.	
	Ins.	*Mm.*		*Ins.*	*Mm.*
Galveston, Tex	15.87	403.10	Staunton, Va	0.74	18.80
Houston, Tex	9.12	231.65	Buffalo, N. Y	0.74	18.80
St. Louis, Mo	8.18	207.77	Parkersburg, W. Va	0.70	17.78
Evansville, Ind	5.60	142.24	Kansas City, Kans	0.70	17.78
Fort Smith, Ark	5.98	151.89	Hartford, Conn	0.68	17.27
Springfield, Ill	4.60	116.84	Pueblo, Colo	0.66	16.76
Memphis, Tenn	4.90	121.46	Omaha, Nebr	0.65	16.51
Shreveport, La	4.82	122.43	North Platte, Nebr	0.64	16.26
Nashville, Tenn	4.54	115.32	Wytheville, Tenn	0.68	17.27
New Orleans, La	4.64	117.86	Montgomery, Ala	0.66	16.76
Harrisburg, Pa	4.38	111.25	Tampa, Fla	0.62	15.75
Burwood, La	3.88	98.55	Asheville, N. C	0.62	15.75
Atlanta, Ga	3.42	86.87	Lynchburg, Va	0.61	15.49
Cairo, Ill	3.34	84.84	Cleveland, Ohio	0.64	16.26
Mobile, Ala	3.34	84.84	Detroit, Mich	0.60	15.24
Canton, N. Y	3.30	83.82	Sioux City, Iowa	0.56	14.22
Chattanooga, Tenn	2.98	75 9	Charlotte, N. C	0.54	13.72
Vicksburg, La	2.79	70.87	Portland, Me	0.52	13.21
Scranton, Pa	2.18	55.37	Meridian, Miss	0.51	12.95
Augusta, Ga	2.56	65.02	Wilmington, N. C	0.48	12.19
Savannah, Ga	2.09	53. 9	Chicago, Ill	0.48	12.19
Peoria, Ill	2.05	52.07	Erie Pa	0.48	12.19
Cincinnati, Ohio	1.98	50.29	Toledo, Ohio	0.44	11.18
Taylor, Tex	1.97	50.04	Norfolk, Va	0 44	11.18
Charleston, S. C	1.93	49.02	Jacksonville, Fla	0.41	10.41
Fort Worth, Tex	1.92	48.77	Grand Rapids, Mich	0.34	8.64
Columbia, S. C	1.86	47.24	Richmond, Va	0.32	8.13
Pensacola, Fla	1.67	42.42	Cheyenne, Wyo	0.26	6.60
Louisville, Ky	1.66	42.16	Key West, Fla	0.25	6.35
Syracuse, N. Y	1.49	37.85	Abilene, Tex	0.21	5.33
Indianapolis, Ind	1.44	36.58	Birmingham, Ala	0.20	5.08
Macon, Ga	1.37	34.80	Lincoln, Nebr	0.18	4.57
Boston, Mass	1.32	33.53	Des Moines, Iowa	0.18	4.57
Lexington, Ky	1.22	30.99	New York, N. Y	0.16	4.06
Oklahoma, Okla	1.20	30.48	Del Rio, Tex	0.11	2.79
Thomasville, Ga	1.20	30.48	Raleigh, N. C	0.06	1.52
Block Island, R. I	1.10	27.94	New Haven, Conn	0.05	1.27
Oswego, N. Y	1.10	27.94	Nantucket, Mass	0.05	1.27
Wichita, Kans	1.08	27.43	Eastport, Me	0.04	1.02
Palestine, Tex	1.06	26.92	San Antonio, Tex	0.03	0.76
Baltimore, Md	0.97	24.64	Dodge City, Kans	0.01	0.25
Albany, N. Y	0.96	24.38	Denver, Colo	0.01	0.25
Northfield, Vt	0.96	24.38	Quebec, Ontario	0.00	0.00
Columbus, Ohio	0.93	23.62	Corpus Christi, Tex	0.00	0.00
Hatteras, N. C	0.92	23.37	Milwaukee, Wis	0.00	0.00
Binghamton, N. Y	0.92	23.37	Alpena, Mich	0.00	0.00
Knoxville, Tenn	0.88	22.35	Philadelphia, Pa	0.00	0.00
Montreal, Quebec	0.88	22.35	Atlantic City, N. J	0.00	0.00

Conclusions.

So far as these five tropical storms are concerned, we can say: (1) That the distribution of rainfall in tropical storms is uniform when compared with the extratropical storms; (2) that the heaviest rainfall usually occurs on that portion of the coast where the storm passes from sea to land; (3) that the velocity of these storms did not decrease as they passed from sea to land; and (4) that the heaviest precipitation usually occurs along the trajectory. In case, however, the storm goes far inland this rule does not hold. In this respect the behavior of the tropical storms resembles that of the extratropical storms. In the study of cyclonic distribution of rainfall in the United States, W. G. Reed [7] found that the area of heaviest precipitation usually occurred on the side which was nearest a large source of moisture.

[6] *Henry, A. J.*, Rivers and floods for August, 1915. MONTHLY WEATHER REVIEW, August, 1915, 43: 413.
[7] *Reed, Wm. Gard.* Study of the cyclonic distribution of rainfall in the United States. MONTHLY WEATHER REVIEW, Oct. 1911, 39: 1609–15. 11 figs.

RADIATION EQUILIBRIUM AND ATMOSPHERIC RADIATION.[1]

By R. EMDEN.

(Abstract and translation by H. BATEMAN, Govans, Md., Aug. 14, 1916.)

1. INTRODUCTION.

Much attention is being paid in the United States to the study of atmospheric radiation and many valuable investigations have been carried out by American meteorologists. These workers have also shown a kindly interest in the researches of European scientists, and as a proof of this the Smithsonian Institution has recently published the admirable work of Anders Ångström in its Miscellaneous Collections (v. 65, no. 3, 1915). An important advance in the theory of atmospheric radiation was made about seven or eight years ago when W. J. Humphreys [2] and E. Gold [3] showed that the sudden decrease of the vertical temperature gradient to a very low value, which is indicated when a balloon rises above the convective region, can be profitably studied in connection with the theory of radiation equilibrium. In Emden's paper the study of this relation is carried further, Humphreys's formula for the temperature of the isothermal region is obtained in another way and Gold's mathematical investigations are repeated and developed under slightly different assumptions. Some of the theoretical results closely resemble those obtained by Gold and are summarized with other interesting conclusions in the latter part of the paper which is printed here in full. The first part of the paper contains a critical survey of the work of Schwarzschild, Gold, and Humphreys and the theory is presented in a clear light. Schwarzschild's method is developed and applied to the case in which the radiation is composed of two parts, each of which can be treated as gray. Emden's work has been summarized by Schmauss in the Meteorologische Zeitschrift (1913, **48**: 454). The same periodical also contains a further contribution to the mathematical analysis by Schwarzschild (Ibid., p. 454), and an article by Gold (Met. Ztschr., 1914, **49**: 89), in which he expresses his views on some of the questions that have been raised with regard to the fundamental hypotheses. As some points are apparently still disputed,[4] a brief résumé of Emden's argument may perhaps be of interest.[5]

2. FUNDAMENTAL ASSUMPTIONS.

(1) In radiation equilibrium each particle of air radiates out just as much energy as it receives from other particles and from external sources if there any. Thus radiation equilibrium obtains when the temperatures of the parts and consequently also the arrangement of the masses are not altered by radiation and absorption. The fundamental condition is that of the equality of the amounts of radiation given up and taken in, irrespective of its composition as regards wave length, state of polarization, and direction.

(2) To simplify the mathematical problem, pressure, density, and temperature are supposed to vary only with the altitude, and the curvature of the level surfaces is neglected. In radiation equilibrium each horizontal layer is supposed to radiate (according to its temperature) just as much energy as it receives from other layers and external sources. *The flow of energy in an upward direc-*

[1] *Emden, Robert*, in Sitzungsb., K. bayerische Akad. d. Wissens., Munchen, 1913, 43:55–142.
[2] Astrophysical Journal (1909).
[3] Proc. Roy. Soc. A. 82, 1909, p. 43.
[4] Cf. Met. Zeitschr., May, 1914, 31:239.
[5] For more recent presentations of the theories of Gold and Humphreys see: *E. Gold.* The International Kite and Balloon Ascents, Geophysical Memoirs, No. 5 (1913). *W. J. Humphreys.* Journal of the Franklin Institute, March (1913).

tion across the layer is, however, not assumed to be equal to the flow of energy downward across the layer.

(3) *The absorption of diffuse radiation can be treated with sufficient exactness by an approximate theory in which diffuse radiation is replaced by parallel radiation and the absorption coefficient for each wave length is doubled.* More precisely, if a horizontal layer of air of mass dm is traversed by a quantity of radiation $S_\lambda \delta\lambda$ with wave lengths lying between λ and $\lambda + \delta\lambda$, the amount absorbed is $a_\lambda S_\lambda \delta\lambda$ for diffuse radiation and $\alpha\lambda S\delta\lambda$ for parallel vertical radiation where $a_\lambda = 2\alpha_\lambda$. The absorption coefficient α_λ is at first assumed to be independent of the pressure which obtains and to be a function only of the wave length λ, but this assumption is abandoned later. The absorption coefficient for diffuse radiation will be denoted by k_λ.

The italic sentences indicate the points in which Emden's hypotheses differ from those of Gold. In the treatment of diffuse radiation Gold endeavors to work with formal rigor. Unfortunately the absorption coefficients of air which come into consideration are not known with sufficient exactness and when values are adopted numerical results can only be obtained after careful mechanical quadratures. Gold also treats solar radiation, earth radiation, and atmospheric radiation separately in his equations and inequalities. In Emden's treatment the two former appear only in the boundary conditions.[6]

Emden adopts the view that the inexactness involved in assumption (3) is negligible compared with that due to our lack of knowledge of the absorption coefficients. To justify this he makes a calculation in which the curvature of rays passing through layers of air of different densities is disregarded[7] and obtains the following results:

1. For black radiation we have accurately $a_\lambda = 2\alpha_\lambda$.
2. If the intensity of the radiation increases in the direction of the masses traversed a_λ lies between α_λ and $2\alpha_\lambda$, if it decreases in the direction of the masses traversed, we have always $a_\lambda > 2\alpha_\lambda$. The variability of a_λ makes a rigorous mathematical treatment of radiation equilibrium very difficult, for a_λ is a function of the distribution of temperature which is to be found. We could calculate it by successive approximations, but in the cases which will be of importance the variation of temperature is such that a_λ is roughly equal to $2\alpha_\lambda$.
3. If a horizontal layer of air at constant temperature is bounded on one side by a black surface at the same temperature, then whatever radiation the layer receives from the black surface it gives it up again to the same amount and with the same wave-length. It thus finds itself in radiation equilibrium. Hence if the earth's surface radiates like a black body, an isothermal atmosphere at the same temperature can not alter the issuing radiation.

3. *Conditions for radiation equilibrium.*

Let E_λ be the emissivity of a black body for wave-length λ and let A_λ, B_λ, be the currents of energy of wave-length λ which cross the boundaries of a thin horizontal layer of mass dm per sq. cm. in the upward and downward directions, respectively, then we have the equations

$$B'_\lambda = k_\lambda E_\lambda - k_\lambda B_\lambda, \tag{1}$$

$$A'_\lambda = k_\lambda A_\lambda - k_\lambda E_\lambda, \tag{2}$$

where primes denote differentiations with respect to m and m is measured vertically downward from the upper limit of the atmosphere. For $m = 0$ the boundary condition is $B_\lambda = \bar{B}_\lambda$, the amount furnished by the solar radiation (diminished by the albedo). At the lower boundary, $m = M$, we have $A_\lambda = \bar{A}_\lambda$, the amount furnished by the earth's radiation. In radiation equilibrium we have for each value of m

$$2\int_0^\infty k_\lambda E_\lambda \, d\lambda = \int_0^\infty k_\lambda \, (B_\lambda + A_\lambda) \, d\lambda, \tag{3}$$

whence $$\int_0^\infty (B_\lambda - A_\lambda) \, d\lambda = \text{constant},$$

and is equal to the difference $\bar{B} - \bar{A}$ between the amounts of energy flowing to and away from the atmosphere at its upper boundary, the so-called "heat balance" (Wärmebilanz).

Let us see under what circumstances an isothermal state can exist. Assuming that E_λ is independent of m we obtain on integration

$$A_\lambda = A_{0\lambda} \, e^{mk_\lambda} + E_\lambda, \qquad B_\lambda = B_{0\lambda} \, e^{-mk_\lambda} + E_\lambda.$$

where $A_{0\lambda}$ and $B_{0\lambda}$ are constants of integration but functions of λ. The condition for radiation equilibrium is that

$$\int_0^\infty (B_{0\lambda} \, e^{-mk_\lambda} - A_{0\lambda} \, e^{mk_\lambda}) \, d\lambda = \text{constant}$$

for all values of m. Emden asserts that this equation can be satisfied only if $B_{0\lambda} = 0$ and $A_{0\lambda} = 0$. In this, however, he appears to be mistaken, for if the absorption coefficient k_λ is a function of λ which is zero when $\lambda = 0$ and $\lambda = \infty$, we can satisfy the requirement by putting

$$A_{0\lambda} = u \frac{d}{d\lambda}(k_\lambda), \qquad B_{0\lambda} = v \frac{d}{d\lambda}(k_\lambda),$$

where u and v are arbitrary constants. It should be noticed that these expressions make $A_{0\lambda}$ and $B_{0\lambda}$ change sign as λ passes through a value for which k_λ is a maximum or minimum. Unless this is physically impossible the conclusion which Emden draws from the equations $A_{0\lambda} = 0$, $B_{0\lambda} = 0$, must be rejected. His conclusion is that $A_\lambda = B_\lambda = E_\lambda$ for all values of m. This means that a portion of the earth's atmosphere can only be in radiation equilibrium at a constant temperature T if it is illuminated from above and below with the radiation from a black body at temperature T; thus according to this conclusion an isothermal layer in radiation equilibrium is impossible under the actual conditions. Influenced by this result Emden obtains an integral equation for the vertical distribution of temperature when there is radiation equilibrium, but as he does not attempt to solve it the equation need not be given here.

4. *Simplification of the problem.*

As very little progress can be made apparently when k_λ varies with λ, Emden simplifies the problem by assuming that the radiation can be divided into two portions (short and long waves), each of which can be treated as "gray radiation," i. e., radiation for which the absorption can be calculated by using an "average absorption coefficient" independent of the wave length. The average absorption coefficients k_1 and k_2 are supposed, however, to be different for the two types of radiation and to be functions of m since they depend on the quantity of water vapor per cubic centimeter of the atmosphere.

[6] Emden uses differential equations while Gold uses integral equations, so that the boundary conditions are contained implicitly in his equations.

[7] He also assumes that the absorption coefficient is independent of the pressure. This assumption is not made in Gold's work.

Writing A and B for the energy currents flowing in the upward and downward directions, respectively, and E for the energy emitted per cubic centimeter by a black body at temperature T, Emden now writes

$$A = A_1 + A_2, \qquad B = B_1 + B_2, \qquad E = E_1 + E_2$$

and adopts the equations

$$B' = k_1 E_1 + k_2 E_2 - k_1 B_1 - k_2 B_2, \tag{1a}$$

$$A' = k_1 A_1 + k_2 A_2 - k_1 E_1 - k_2 E_2, \tag{2a}$$

$$2(k_1 E_1 + k_2 E_2) = k_1 B_1 + k_2 B_2 + k_1 A_1 + k_2 A_2, \tag{3a}$$

as the analogues of (1), (2), and (3).

The absorption coefficients of each layer of air for each type of radiation are now assumed to be proportional to the water vapor content, and by using well-known empirical formulæ for the variation of density and humidity with the height Emden is led to adopt the expressions—

$$k_1 = 4b_1 m^3 = 0.4m^3, \qquad k_2 = 4b_2 m^3 = 9.2m^3.$$

Emden now neglects A_1 in comparison with A_2; k_1A_1, in comparison with k_2A_2; and E_1 in comparison with E_2. Then, assuming that for $m=0$, $B-A=0$, so that the 'heat balance' is zero, he obtains from the above relations and from $B'_1 = -k_1 B_1$

$$B = A = \sigma\left\{\frac{b_2+b_1}{2b_1} - \frac{b_2-b_1}{2b_1}\,e^{-b_1 m^4}\right\} \tag{4}$$

$$E = \sigma\frac{b_1+b_2}{2b_1 b_2}\left\{b_2 - (b_2-b_1)\,e^{-b_1 m^4}\right\} \tag{5}$$

where σ is the fourth part of the solar constant diminished by the albedo. Putting $E = sT^4$, $\sigma = s\tau^4$, where $\tau = 254°$, the 'effective earth temperature,' we obtain the value of T. At a considerable height m is small and we have Emden's equation (86)

$$T = 254°\left(\tfrac{1}{2} + \frac{b_1}{2b_2}\right)^{\frac{1}{4}} = 254°\left(\tfrac{1}{2} + \frac{k_1}{2k_2}\right)^{\frac{1}{4}}.$$

Some values of T are given in Table 1.

TABLE 1.—*Values of T for various values of the ratio k_1/k_2.*

$\frac{k_1}{k_2}$	1	$\frac{1}{2}$	$\frac{1}{5}$	$\frac{1}{10}$	$\frac{1}{23}$	0
T....................	254°A −19°C	238°A −35°C	224°A −49°C	219°A −54°C	215.87°A −57.23°C	213.7°A −59.3°C

The temperature −57.23° C is called the "inversion temperature" and denoted by T_i. The value obtained for $\frac{k_1}{k_2}=0$ agrees with Humphreys' value. Above a certain level we may put $p = gm$ in (5) and we then find with the aid of the relations $dp = g\rho dz$, $p = g\rho RT$, that the temperature gradient is given by Emden's (90)

$$\frac{dT}{dz} = \frac{1}{R}\frac{T^4 - T_i^4}{T^4} \tag{6}$$

For stability we must have

$$\frac{dT}{dz} < \frac{1}{R}\frac{\kappa - 1}{\kappa},$$

where κ is the ratio of the specific heats = 7/5. If the atmosphere is in radiation equilibrium only those layers are stable for which

$$7T^4 < 5T_i^4 \text{ A. or } T < 234.8° = -38.2°\text{C}.$$

The variation of temperature with height may be derived from (6) and Emden obtains the following results: The height h is measured in meters:

TABLE 2.—*Vertical temperature changes by (6).*

T............	215.84°	216°	216.1°	216.2°	216.3°	220°	230°
$-\frac{dT}{dh}$.......	0	0.000081	0.00014	0.00021	0.00029	0.00249	0.00765
h............	00	11500	10530	9960	9540	5830	3770
T............	234.82°	240°	250°	260°	270°	280°	288.8° A
$-\frac{dT}{dh}$.......	0.0098	0.0118	0.0152	0.0178	0.0203	0.0221	0.0235
h............	3130	2730	1990	1380	860	386	0

Emden also calculates from equations (4) and (5) that the temperature of the lowest atmospheric layer is 15.8°C and that the temperature of the ground is 36°C. The discontinuity in temperature of 20 degrees is in reality greatly diminished by conduction of heat and evaporation. The remainder of the paper is now given in full.

5. *Summary.*

Let us consider briefly the foundations and results of the theory that has just been developed. If the radiation coming from the sun and amounting to 2 gram. cal./cm². min. were distributed uniformly over the surface of the solid earth deprived of its atmosphere, then on the assumption of gray radiation the earth would, independently of its emission (absorption capacity) set itself at the temperature of radiation equilibrium, viz, $T = 285° = 12°$C. Taking into account the albedo of the earth, the inflow of radiation is diminished by 37 per cent, consequently this temperature is lowered to $T = 254° = -19°$C. We call this temperature the *effective earth temperature*. If now we cover the earth with an arbitrary atmosphere, then whether its layers are arbitrarily poor or rich in water vapor and absorb weakly or strongly, it will set itself in radiation equilibrium, with its lower part isothermal, at the effective earth temperature $T = 254° = -19°$C (in agreement with Gold), provided the radiation is gray, i. e., provided the absorption capacity is assumed to be the same arbitrary quantity for all wave-lengths. If we examine the behavior for separate wave-lengths, we are led to an integral equation which can not be dealt with on account of the lack of the necessary physical data. We have consequently adopted an intermediary method in which the short-wave radiation of the sun and the long-wave counter radiation of the earth are each assumed to constitute gray radiation but with different absorption coefficients. The absorption (emission) of each atmospheric layer is put proportional to its water-vapor content; it is small for the incident short-wave radiation, large for the returning long-wave radiation; only when the water-vapor content is very small do they approach a common very small value. On this basis the following result was obtained:

Above a certain level the incident radiation is only weakened a little and the temperature of radiation equilibrium for each layer is determined solely by the ratio in which its content of water vapor absorbs short-waved and long-waved radiation, independently of the absolute values. When this ratio is zero we obtain the minimum temperature at which radiation equilibrium can occur, viz., $2\frac{1}{4}$ times smaller than the effective earth temperature, or 213.7° (−59.3°C.). For average humidity conditions we must diminish this ratio by $\frac{1}{23}$ giving a temperature of −57° C.; the value $\frac{1}{10}$ would give −54°C. These are the known temperatures of the stratosphere. In ascending the ratio $\frac{k_1}{k_2}$ increases with decreasing water-vapor content, the temperatures rise attaining the effective earth temperature for $\frac{k_1}{k_2}=1$. If the highest layers of the atmosphere are, as is to be expected, sufficiently poor in water vapor, then in spite of wide-spread ideas to the contrary, they are not characterized by very low temperatures, but find themselves throughout at temperature −19°C. At this temperature the atmosphere passes isothermally with increasing attenuation, through conditions which on account of the lack of physical data we are not able to deal with more closely, into the so-called empty space. If, on the other hand, we go downward, the temperatures likewise increase with increasing absorption. Since now, however, the absolute values of the absorption capacities of water vapor for short waves and long waves come into action, the calculated temperatures are essentially uncertain. Nevertheless we are able to calculate the temperature of the ground layer of the atmosphere as 15.8°C., agreeing well with observation. The important result was thereby obtained that from a level of about 3,000m. downward the atmosphere is mechanically unstable in radiation equilibrium. Convection will invalidate the calculated temperatures, but the temperature above this region and the temperatures of the ground layers will vary very little.

Now it is known that the atmosphere below an average height of about 10km. is almost always traversed vertically by the convection currents of long cycles, the characteristic phenomena of high and low pressure regions, as well as by those convection currents in which the general circulation of the atmosphere clearly enters into the phenomena. Here, again, the radiation temperatures above the prescribed level are not altered; those of the ground layers very little. Above this level, however, the absorption of the incident radiation is so small that the temperatures can be calculated from Table 1. We thus obtain with any desired exactness the temperatures of the stratosphere given by observation as temperatures of radiation equilibrium of these layers, determined by the ratio with which the water-vapor content absorbs the short and long wave radiation. The radiation theory furnishes the temperature of the stratosphere with any desired exactness. It gives a complete explanation of the thermal discontinuity which is connected with the mechanical discontinuity in the division of the atmosphere into troposphere and stratosphere. The radiation theory can not explain the mechanical discontinuity in the first place; nevertheless the possibility of establishing it is not excluded. To see this, we first of all make a distinction between our results and Gold's second result. If gray radiation is postulated, as with Gold,[8] the atmosphere is always arranged in a stable fashion by radiation, the absorption being an arbitrary function of the height. Starting from a convective state a separation of the atmosphere at $m=\frac{M}{4}$ can indeed be transitorily produced, so that convection currents are created in the lower part. But the final product is always stability; if the heat balance is null it is the isothermal state. On the other hand, our theory indicates that by radiation alone instability is continually produced in the lowest 3 kilometers of the atmosphere; in the lowest parts it is of such great intensity that a perpetual cause of convection currents is present. Consequently in an atmosphere which is not influenced by the general circulation a separation corresponding to the troposphere and stratosphere must occur on account of radiation alone at the height to which the convection currents extend. This height would be given by the condition that the underlying layers, intermixed by convection, send out just as much radiation as in radiation equilibrium; and it must be shown that the convection currents associated with instability can drive up to this height. It is not impossible that an admissible height will thereby be obtained, but then it will be uncertain and suspicious to ascribe the division of the atmosphere to these processes of radiation. Indeed the general circulation can surpass radiation in action determining temperature, and in the following paragraphs we shall consider that the air masses of temperate latitudes receive their radiating capacity not on the spot but, in fact, in the winter in equatorial regions, and have carried it to us from there. The induced instability, however, deserves close consideration in all cases, particularly in its action in regions and at times which are characterized by "radiation weather."

The calculated temperatures are proportional to the effective earth temperatures, the fourth powers of which are determined by the solar constant and the albedo of the earth. The first quantity is measured with sufficient exactness; a variation of the albedo by 10 per cent of its value would alter the effective earth temperature by about $4\frac{1}{2}$°, the inversion temperature, and thereby the temperatures of the stratosphere by about $3\frac{1}{2}$°. The agreement with observation would still be advantageous if we consider that we can compensate temperatures that are too low by a decrease in the water-vapor content of the stratosphere. It appears to me that a more suspicous simplification is that of spreading the flow of energy uniformly over the whole surface of the earth whereby this amounts to $\frac{2\times 0.63}{4}=0.315\ \frac{\text{gram cal}}{\text{min}}$ (albedo 0.37) leading to the effective earth temperature 254° = −19°C. For the yearly sum of the solar radiations for latitude 50 measures only 70 per cent and for the pole only 42 per cent of the value at the Equator. Even if we disregard the polar caps, the higher latitudes would, with the assumed distribution, receive too much, in case the geometrical conditions of the radiation were alone important in determining temperature. That it is not permissible, however, to introduce the effective temperatures for different parallels of latitude according to the amounts of radiation they receive is shown immediately by an attempt to take into account the influence of the seasons.

Thus in the summer half-year latitude 50° receives an amount of solar radiation represented by the number 183, in winter by 67 equatorial days,[9] for latitude 60° we get the numbers 169.5 and 38. The effective earth tempera-

[8] This statement is misleading, as Gold does not postulate gray radiation. The above is Emden's own interpretation of Gold's result.—*H. Bateman.*

[9] *Hann, J.* Handbuch der Klimatologie, Stuttgart, 1908, v. 1, p. 100.

tures (and the calculated radiation temperatures proportional to these) must behave like the 4th roots, for latitude 50° like 1 : 1.25, for latitude 60° like 1 : 1.45. The summer and winter temperatures of these parallels of latitude are 291.1°(18.1°C.) and 266°(−7°C.), respectively, and 287°(14°C.) and 257.2°(−15.8°C.), respectively, they thus behave only like 1:1.09 and 1:1.12, respectively. The temperatures of higher latitudes are thus not generated by solar radiation at the place. The solar radiation indeed governs the average yearly temperature of the earth as a whole; its variation with latitude and the deviations from a mean value during the year are determined by other factors. The general circulation of the atmosphere provides quantities of entropy to higher latitudes by the transportation of masses of air at higher temperatures, and in the same measure as it regulates the compensation of temperatures as the time of the year changes, it takes care also of the uniform distribution of temperature which lies at the basis of our theory. The temperatures of the lower layers of the stratosphere are, however, governed by the radiation of the deeper layers of the troposphere (we would otherwise meet with temperatures equal to the effective earth temperature); the general circulation consequently also hinders the otherwise strongly local and seasonal oscillation of the inversion temperature. Wagner's investigations [10] show, however, that the time of the year still has some effect, for the maximum temperature of the stratosphere in June was −52°C, the minimum temperature in January was −61.4°C. The oscillation of 9.4 degrees is smaller than the annual oscillation at the earth's surface. Our theory gives a satisfactory explanation. The atmospheric strata, and thus also the lower layers of the stratosphere, are richer in water vapor in the summer; the smaller the water-vapor content, the higher is the temperature of these layers. (See Table 1.) The lower temperature of the radiating layers of the troposphere is partly compensated by smaller water-vapor content at the upper inversion. The investigations of the following will give further insight into these conditions.

6. THE RADIATION OF THE ATMOSPHERE.

The investigations of the last paragraph determine the temperature of radiation equilibrium of the atmosphere in the case when the heat balance is given. In this paragraph we treat the converse problem. Let the temperature of the atmosphere and the outer radiation be given. What is the radiation which traverses each cross-section of the atmosphere and emerges at its boundary? We start from the same assumptions as before but E is now a given function of m and the equations may be integrated, giving

$$B_1 = \sigma e^{-b_1 m^4} \text{ and } B_2 = \overline{B}_2 e^{-b_2 m^4} + 4 b_2 e^{-b_2 m^4} \int_0^m m^3 e^{b_2 m^4} E \, dm.$$

Taking into consideration the fact that $\overline{B}_2$ (the value of B at the upper limit of the atmosphere) is zero, we finally obtain after a simple transformation

$$B = \sigma\, e^{-b_1 m^4} - \overline{E}\, e^{-b_2 m^4} + E - e^{-b_2 m^4} \int_0^m e^{b_2 m^4} E' dm, \qquad (7)$$

where $\overline{E}$ denotes the value of E at the upper limit of the atmosphere. Likewise we find

$$A = \overline{A} e^{b_2 m^4} - 4 b_2 e^{b_2 m^4} \int_0^m m^3 e^{-b_2 m^4} E \, dm,$$

and after a simple transformation

$$A = (\overline{A} - \overline{E})\, e^{b_2 m^4} + E - e^{b_2 m^4} \int_0^m e^{-b_2 m^4} E' \, dm. \qquad (8)$$

If instead of the values $\overline{A}$ and $\overline{E}$ at the upper limit we introduce the values $\underline{A}$ and $\underline{E}$ at the lower limit of the atmosphere ($m = M$) we obtain

$$A = (\underline{A} - \underline{E})\, e^{b_2 (m^4 - M^4)} + E + e^{b_2 m^4} \int_m^M e^{-b_2 m^4} E' \, dm. \qquad (9)$$

Equations (7), (8), and (9) contain the complete solution of our problem.

If now we calculate the quantity

$$(2 k_2 E - k_1 B_1 - k_2 B - k_2 A)\, dm \qquad (10)$$

we obtain the yield of energy for each layer and consequently the velocity of cooling. Integration with respect to m gives the yield of heat from atmospheric layers of finite thickness.

Applications of the above equations.

We first of all give a table of the temperatures at which a black body radiates certain quantities of energy, E, measured in gram-calories per square centimeter per minute. This unit will be denoted in future by G.

TABLE 3.—*Temperatures at which a black body radiates the energy E.*

T......	210°A	230°	240°	250°	260°	270°	280°	285°	290°	300°	310°	340°
E......	0.148 gr.-cal.	0.212	0.252	0.296	0.347	0.403	0.467	0.500	0.537	0.615	0.701	1.014

(*1*) *The nocturnal cooling of an isothermal atmosphere.*—Let the temperature be T. In consequence of the constancy of temperature, E is also constant, therefore $E' = 0$ and so $\overline{E} = \underline{E} = E$. Taking into account the nocturnal conditions we put $\sigma = 0$ in (7) and obtain

$$B = E(1 - e^{-b_2 m^4}) \qquad (11)$$

$$A = (\underline{A} - E) e^{-b_2 (m^4 - M^4)} + E \qquad (12)$$

Since we wish to discuss the radiation of only the atmosphere itself, we assume further that the earth's surface is likewise at the same temperature and radiates like a black body, or finds itself at a somewhat higher temperature with a smaller capacity for radiation. so that we have $\underline{A} = E$ and we obtain

$$A = E \qquad (13)$$

The radiation B, which is not so important for our investigation, is uninfluenced by this simplification. Equation (13) again gives the theorem that an isothermal atmosphere does not alter the radiation of a black body at the same temperature. A horizontal surface reflecting on its upper surface and black on its lower surface would measure, at a place in the atmosphere, a radiation independent of the height and equal in intensity to the earth's radiation. When turned over the irradiated surface measures the radiation directed earthward which is given by (11). The factor $1 - e^{-b_2 m^4}$ is, as is easily seen, the absorption capacity of the layers lying above the level m, so that equation (11) is the expression of Kirchhoff's law, which holds here unchanged, since we assume

[10] *Wagner, A.* Die Temperaturverhältnisse in der freien Atmosphäre. Beitr. z. Physik d. fr. Atmosph., 1910, 3:57.
Dines, W. H., in Phil. trans., 1911, 211A:253.

an isothermal radiator. For the ground layer, $m=1$, we have $e^{-b_2m^4}=0.1$ with average humidity conditions, so that the earth's surface receives, radiated to it from the atmosphere, a quantity of heat equal to 90 per cent of the quantity radiated by a black body at the same temperature! With increasing radiation the inflow of heat to the atmosphere decreases rapidly as the following table shows. The radiation is measured in G's as before.

TABLE 4.—*Inflow of heat to the atmosphere at different levels and temperatures.*

h	T				
	−20° C.	−10° C.	0° C.	10° C.	20° C.
Meters.	G.	G.	G.	G.	G.
0	0.28	0.32	0.38	0.44	0.50
1,000	0.23	0.27	0.315	0.37	0.42
2,000	0.18	0.21	0.23	0.28	0.32
3,000	0.12	0.14	0.16	0.19	0.22
4,000	0.08	0.10	0.11	0.13	0.15
5,540	0.04	0.05	0.055	0.06	0.07

Direct measurements [11] give—

Observed temperature.	Naples (60m).	Vienna (220m).	Zurich (440m).	Rawics (950m).	Sonnblick (3,100m).	
	22° C.	19°	15°	−6°	−1°	−12°
Counter radiation of the atmosphere	0.40	0.41	0.37	0.21	0.23	0.12

The differences (calculated − observed) are with one exception positive, decreasing with increasing height, the reason being that the calculations are based on temperatures which do not vary with the height.

In comparison with the previous numbers representing the relation between E and T, the numbers show how great the protecting effect (Strahlungsschutz) of an isothermal atmosphere is under ordinary humidity conditions. At sealevel the counter-radiation of this atmosphere amounts to 90 per cent of the outer radiation of the earth's surface when the latter has its maximum value (black-body radiation); it can thus keep the earth's loss of heat as low as 10 per cent. The extraordinary influence of the water vapor is also shown, for the decrease of the numbers with the height is to be ascribed chiefly to the water vapor.

To calculate the quantity of heat, dQ, given up by an atmospheric layer we find from (10) that

$$dQ=4Ee^{-b_2m^4}b_2m^3dm, \qquad (14)$$

and so

$$Q=E\,(e^{-b_2m_1^4}-e^{-b_2m_2^4}). \qquad (15)$$

Consequently the layers lying above m give a quantity of heat

$$Q=E\,(1-e^{-b_2m^4}),$$

which is the quantity furnished by the radiation B. The heat given up by this layer thus wanders only in the direction of the earth. This is evident because the constant energy current A, which leaves the atmosphere in the direction of space, enters at the lower surface.

The temperature of each layer falls in a minute by the quantity

$$\Delta T=\frac{4E}{c_p}e^{-b_2m^4}b_2m^3, \qquad (16)$$

and the mean temperature of the layer from m_1 to m_2 by

$$\Delta T=\frac{E}{c_p(m_2-m_1)}(e^{-b_2m_1^4}-e^{-b_2m_2^4}). \qquad (17)$$

The temperature does not fall at the same rate in all layers. The isothermal condition will consequently cease on account of the process of radiation, but very slowly. The layer $m^4=3/4b_2$ suffers the quickest change of temperature; this is at a height of about 2,250 m; with $T=270°$ its temperature sinks by 0.00323° per minute, and 5 hours 10 minutes are required for 1° of cooling.

To cool by 1° on the average, the lowest layer of 1 km. thickness requires 7 hours 20 minutes, the whole atmosphere 10 hours 45 minutes. The radiation processes of one night may consequently not alter an isothermal atmosphere in an essential way, the lowest dust-laden layers excepted: the low temperatures of the stratosphere do not react in a more appreciable way at the times of transition between night and day.

II. THE NOCTURNAL RADIATION OF A POLYTROPIC ATMOSPHERE.[12]

Using the same boundary conditions as in the last case we put $\sigma=0$ and $\underline{A}=\underline{E}$, since in any polytropic atmosphere the temperature decreases linearly with the height when q is constant $\bar{E}$ will be zero; equations (7) and (8) now give

$$B=E-e^{-b_2m^4}\int_0^m e^{b_2m^4}E'dm, \quad A=E+e^{b_2m^4}\int_m^M e^{-b_2m^4}E'dm.$$

To simplify the calculation as much as possible we put

$$E=E_0\frac{m}{M}, \qquad M=1.$$

Since $m\sim p$ and $E\sim T^4$ we have the polytrope

$$\frac{T}{T_0}=\left(\frac{p}{p_0}\right)^{\frac{\kappa-1}{\kappa}}=\left(\frac{p}{p_0}\right)^{\frac{1}{4}}$$

thus $\kappa=\frac{4}{3}$; this gives the fall of temperature

$$\frac{dT}{dh}=-\frac{\kappa-1}{\kappa R}=-\frac{0.0085°}{\text{meter}}.$$

The temperature decreases by 0.85° per 100 meters, thus exceptionally rapidly when we take into account the above exceptionally small fall of temperature. We can thus estimate the radiation under average conditions. In paragraph 3 of his paper Gold has assumed the same temperature gradients. We are thus in a position to be able to compare his results with our theory. We thus obtain

$$B=E_0\left[m-e^{-b_2m^4}\int_0^m e^{b_2m^4}dm\right]. \qquad (18)$$

$$A=E_0\left[m+e^{b_2m^4}\int_m^M e^{-b_2m^4}dm\right]. \qquad (19)$$

The integrals can be evaluated by development in series which, since $b_2=2.3$, converge slowly for the maxi-

[11] *Trabert, W.* Lehrbuch der kosmische Physik. Leipzig, etc., 1911, p. 456.

[12] *Emden.* Gaskugeln. Leipzig, 1907. Chapter 17, §2.
The term "polytropic atmosphere" is used by Emden to denote an atmosphere or portion of an atmosphere in which the pressure, absolute temperature, and specific volume are connected by the relations

$Tv^{\kappa-1}$=constant; pv^{κ}=constant.

mum value $m = M = 1$, and the 10th term must be taken into account.

TABLE 5.—*The counter radiation B for different bottom temperatures, T_0, and altitudes, h.*

[Gram-calories per sq. cm., per minute.]

h	T_0 (air temperature at the ground).					
	−20°	−10°	0°	10°	20°	30°
M.	*Gr.-cal.*	*Gr.-cal.*	*Gr.-cal.*	*Gr.-cal.*	*Gr.-cal.*	*Gr.-cal.*
0	0.26	0.29	0.335	0.39	0.45	0.51
1,000	0.18	0.21	0.27	0.28	0.32	0.36
2,000	0.12	0.14	0.155	0.18	0.21	0.24
3,000	0.07	0.08	0.09	0.11	0.13	0.14
4,000	0.04	0.05	0.05	0.065	0.07	0.08
5,540	0.015	0.02	0.02	0.03	0.03	0.03

The quantities of radiation radiated to the atmosphere are evidently smaller as in the case of an isothermal atmosphere, for $\underline{B}$ we obtain $0.798E_0$ instead of $0.9E$.

For the sake of comparison we give the few available observations and add the temperatures T_0 corresponding to the assumed gradient.

TABLE 6.—*Comparison of observed and computed temperatures and counter-radiations.*

	Naples (60 m.)	Vienna (220 m.)	Zurich (440 m.)	Rauris (950 m.)	Sonnblick (3,100 m.)	
Observed temperature	22°	19°	15°	−6°	−1°	−12°
Calculated to	22°	21°	19°	2°	25°	14°
Observed counter-radiation	0.40	0.41	0.37	0.21	0.23	0.12
Observed calculated isothermally	0.50	0.48	0.44	0.29	0.16	0.14
Observed calculated polytropically	0.46	0.42	0.38	0.24	0.14	0.12

The agreement with the observed values is as good as can be expected considering the difficulty of the measurements and the average conditions at the basis of the calculation. The disagreement of the measurement 0.23 on Sonnblick is an exception; in comparison with the other measurements, this value appears exceptionally large. *The agreement shows that with our simplifying assumption of dividing the radiation into short waves and long waves we form a very good estimate of the conditions actually obtaining.*

If, in order to approach closer to average conditions, we choose the exponent $\kappa = 7/6$ of the polytrope, we should obtain the temperature gradient 0.49° per 100 meters, and we should have to evaluate integrals of the form

$$\int_0^m e^{\pm b_2 m^4} m^{-\frac{1}{7}}\, dm.$$

For an atmosphere of this structure, we obtain $B = 0.838E_0$ instead of the values $0.798E_0$ and $0.9E_0$ just calculated. The two tables thus suffice for a judgment of the average conditions.

The radiation A can be calculated for each level from (19). In a free balloon at night the radiation $B + A$ would be obtained per horizontal square centimeter; it can serve as a measure of radiation.

For the radiation $\bar{A}$ which the atmosphere loses at its upper boundary, we obtain $\bar{A} = 0.7274E_0$, thus 73 per cent of the quantity which enters the atmosphere below. (For the temperature gradient 0.49 we get $\bar{A} = 0.826E_0$.) The atmosphere's gain in radiation is thus

$$E_0\,(1 - 0.7274 - 0.798) = -0.525E_0$$

gram-cal. per sq. cm. per minute.

It will consequently cool by about 0.00090 degree a minute, or by 1 degree in about 18 hours 30 minutes, if the ground temperature is 0°C. The time of cooling of the lowest kilometer is evidently smaller than 7 hours 20 minutes, the isothermal value. The cooling during the night is extraordinarily small.

To find the yield of heat dQ, we use (10) and (13) and obtain

$$dQ = \left[e^{-b_2 m^4}\int_0^m e^{b_2 m^4}\, dm - e^{b_2 m}\int_m^M e^{-b_2 m^4}\, dm\right] 4b_2\, m^3\, dm, \qquad (20)$$

and for a finite layer from m_1 to m_2

$$Q = e^{-b_2 m_1^4}\int_0^{m_1} e^{b_2 m^4}\, dm - e^{-b_2 m_2^4}\int_0^{m_2} e^{b_2 m^4}\, dm$$

$$+ e^{b_2 m_1^4}\int_{m_1}^M e^{-b_2 m^4}\, dm - e^{b_2 m_2^4}\int_m^M e^{b_2 m^4}\, dm,$$

where the quantity already calculated is again given for $m_1 = 0$, $m_2 = M$. In (20) $dQ > 0$ when m is large. For small values of m, we expand in powers of m and neglect m^4 and higher powers. The quantity within square brackets then becomes

$$2m - (1 - \tfrac{1}{5}b_2).$$

Thus

$$dQ \lesseqgtr 0 \text{ according as } m \lesseqgtr 0.27.$$

In all layers for which $m \lesseqgtr 0.27$ the outward radiation is $\lesseqgtr$ the absorption. The layers higher than $m = 0.27$ are warmed and the lower ones cooled by the radiation process. Let us calculate the height of this layer.

Since the temperature gradient is given by

$$\frac{dT}{dh} = -\frac{\kappa - 1}{\kappa R}$$

the polytrope

$$\frac{T}{T_0} = \left(\frac{p}{p_0}\right)^{\frac{\kappa-1}{\kappa}}$$

gives

$$dT = T_0 dp^{\frac{\kappa-1}{\kappa}}$$

if we put

$$p_0 = M = 1 \text{ and } p = m^{\frac{\kappa-1}{\kappa}}$$

We then obtain

$$1 - m^{\frac{\kappa-1}{\kappa}} = \frac{h(\kappa - 1)}{\kappa R T_0}.$$

Using our value $\kappa = 4/3$ and $m = 0.27$, we get

$$1 - m^{\frac{\kappa-1}{\kappa}} = 0.28 \text{ and for } T_0 = 273,$$

$$h = 8{,}950 \text{ meters.}$$

For the temperature gradient $0.49°/m$ we should have obtained 8,200 m.; for $0.1°/m$, 8,650 m.; increasing to ∞ for an isothermal atmosphere. Instead of our value $m = 0.27$. Gold found 0.25 under different assumptions as to the distribution of water vapor. Above this height a polytropic atmosphere would be warmed by the process of radiation; below it would be cooled.

III. ATMOSPHERIC RADIATION AND SOLAR RADIATION.

In order to free our ideas on the heating of the earth from many conventional obscurities, we ask the question: *Can the quantity of radiation which the earth's surface receives be increased by an absorbing atmosphere placed between the sun and the earth.* The answer is in the affirmative. Let us put the question in another form: Would the mean temperature of the earth's surface rise or fall if the atmosphere were taken away. At higher temperatures, i. e., with stronger outward radiation, more heat must also be conveyed to it. If the atmosphere justifies the idea that it is a heat conserver, thus increasing the temperature, it must augment the radiation.

The fact that an absorbing substance can strengthen the radiation passing through it appears paradoxical to us, since we involuntarily call to mind the laboratory experiments by which we determine absorption coefficients. The absorbing substance is interposed between the source of radiation and the receiving measuring surface. In this it is essential that the absorbing substance and measuring surface should be at a lower temperature than the source of radiation, so that only the weakening of the radiation is determined. With increasing temperature of the absorbing substance the conditions are completely changed. If for instance we take as a source of radiation a black glowing surface and interpose a cold gas, the dark absorption lines appear; with a hotter gas, the emission preponderates in spite of the absorption and the lines appear bright. If, on the other hand, we take care that the radiating and absorbing (receiving) bodies can only exchange heat among themselves by radiation, then radiation equilibrium for this system sets in and every absorbing body emits black radiation.

The earth's surface without an atmosphere would set itself at the effective earth temperature if the sun's radiation were uniformly distributed. If we again add the atmosphere, assume gray radiation, and await radiation equilibrium, the atmosphere sets itself isothermally at the same temperature and each layer is traversed upward and downward by the same energy current independently of its height. The radiation which the earth's surface receives remains unaltered, and the "heat conservation (Wärmeschutz)" of this atmosphere would be null. Strictly speaking, it would be negative. Diffuse reflection and clouds diminish the useful radiation by 37 per cent, the value of the albedo; but we omit this quantity of energy in practice, since it does not enter into the thermodynamical system and is always left out of consideration.

If we abandon the hypothesis of gray radiation, which leads to unsatisfactory results, and again divide the radiation into short and long waves, the earth's surface is met in radiation equilibrium by radiation B determined by equation (4) of paragraph 4. Denoting by σ the radiation incident on the upper limit of the atmosphere we get

$$\underline{B}=\sigma(2.2)^{\frac{1}{4}}=1.218\sigma.$$

There is conservation of heat; the interposed atmosphere strengthens the incident radiation by about 22 per cent. The mechanism of the radiation process is clear. The atmosphere is warmed not only by the radiation incident from above, σ, but chiefly by the counter-radiation from the earth's surface, and this counter-radiation is itself governed by the solar radiation and the counter-radiation of the atmosphere. The radiations $\underline{B}=\underline{A}=1.218\sigma$ are diminished by the absorption and emission of the atmosphere to the values $\overline{B}=\overline{A}=\sigma$ at the upper boundary, so that the balance of heat is null for the whole.

The convection currents of long cycles hinder the formation of radiation equilibrium within the troposphere. The temperatures of the stratosphere are not altered thereby; its low temperature and small content in water vapor permits it to send out only small quantities of radiation. The counter-radiation of the atmosphere is governed almost exclusively by the condition of the troposphere, particularly in the layers near the earth. Consequently we come very close to average conditions when we assume a decrease of temperature of 0.5° per 100 m.; the temperature conditions of the lower important layers are sufficiently well represented thereby. The counter-radiation is estimated with sufficient exactness as

$$\underline{B}=0.84\underline{E}_0=0.84s\,\underline{T}_0^4, \qquad (21)$$

where T_0 measures the ground temperature of the atmosphere. In order to take into account as far as possible the conditions actually obtaining, we abandon the assumption that the solar radiation is distributed uniformly over the earth's surface; that this is throughout at an average temperature. We make an investigation taking into account the geographical latitude. Accordingly we put for T_0 in (21) the mean temperature of the parallel of latitude, using Spitaler's values for the year, for July, the warmest month, and for January, the coldest month. We thereby calculate the counter-radiation (B) of the atmosphere and compare it with the radiation which the sun sends to this parallel of latitude with the full quantity of radiation $\sigma=2\,G$'s *without subtracting the albedo.* (It would be too troublesome and uncertain to calculate the albedo of each parallel of latitude.) The results of the calculation are given in Table 7.

TABLE 7.—*Solar radiation and the atmospheric counter radiation for certain parallels of the Northern Hemisphere.* (*Emden, p. 136, Table II.*)[13]

[Gram-calories per square centimeter per 24 hours.]

North latitude.	Temperatures.			Mean annual solar radiation.		Mean annual counter radiation.
	Annual.	July.	January.			
	1	2	3	4	5	6
	°C.	°C.	°C.	*Equatorial days.*	*Gram-calories.*	*Gram-calories.*
0°	25.9	25.5	26.2	365	880	733
15°	26.3	27.9	23.9	354	852	736
20°	25.6	28.1	21.7	345	830	732
30°	20.3	27.4	13.9	321	773	681
40°	14.0	23.8	3.9	288.5	694	624
50°	5.6	18.1	−7.2	250	601	554
60°	−0.1	14.1	−16.0	208	500	510

North latitude.	Solar radiation, June 21.		Counter radiation, July.	Solar radiation, Dec. 21.		Counter radiation, January.
	7	8	9	10	11	12
	Equatorial days.	*Gram-calories.*	*Gram-calories.*	*Equatorial days.*	*Gram-calories.*	*Gram-calories.*
0°	381	917	730	358	863	736
15°	387	933.5	753	285	687	713
20°	398	958	755	258	621	694
30°	411	998	750	198	477	622
40°	421.5	1,015	713	135	326	540
50°	421	1,015	660	75	181	460
60°	416	1,002	624	21	51	403

[13] The quantities of solar radiation in equatorial days are taken from *Hann*, Handbuch der Klimatologie, Stuttgart, 1908. Bd. I, p. 94. At the time when day and night are equal the equator receives $1440\sigma/\pi=458.4\sigma$ gram. cal./cm², 24 hrs.; the wandering of the sun between the tropics (Wendekreise) makes this quantity 0.9592 times smaller on the yearly average. An equatorial day thus corresponds to a supply of heat equal to $0.9592\times458.4\sigma=439.7\sigma=879.4$ gram-cal./cm²., 24 hrs.

We first consider the mean yearly conditions. In the equatorial regions themselves the counter radiation of the atmosphere is only about 20 to 10 per cent smaller than the inflow of radiation at the upper boundary. Since the content of the atmosphere in water vapor is certainly greater than the assumed mean value in these regions, the counter radiation is still greater. (Since we put $B = 0.84 s T_0^4$, it can be always about 16 per cent greater.) Between latitudes 50° and 60° the two radiations are already equal to one another. A considerable part of the calculated solar radiation is, however, lost for radiation to the earth by reflection at the clouds and by diffuse reflection. If we estimate the diffusely reflected portion at 19 per cent, as usual, the solar radiation will be already reduced below the atmospheric radiation. We thus have the theorem:

The annual counter radiation of the atmosphere is only a little smaller than the annual solar radiation which the atmosphere encounters at its outer limit, and is greater than the annual solar radiation which arrives at the solid earth.

If we reduce the given value of the solar radiation by 37 per cent, the mean value of the albedo, we obtain between latitudes 0° and 30°—or for half the earth's surface—about $840 \times 0.63 = 530$cal.: in the assumed polytropic atmosphere the outward radiation of earth + atmosphere would be almost exactly equal to the given values of the counter radiation; for on page 22 we found the gradient 0.49°/100m to give $B = 0.84 E_0$, and $A = 0.83 E_0$.

The heat balance would thus no longer be zero, since in the course of the year more heat would be radiated outward than inward. But the assumed gradient holds only for the lower layers of the atmosphere which chiefly determine the counter radiation. By assuming this gradient we have ascribed far too high temperatures to the upper layers which govern the upward radiation; at heights of 10 kilometers we would obtain −25° to −30°C instead of −50° to −57°, the temperatures of the stratosphere. The outward radiation consequently turns out to be much smaller.

If we calculate with the radiation of the stratosphere in radiation equilibrium, we obtain instead of an average of about 720 cal./cm² 24 hrs. of counter radiation, the amount $1440 \times 0.315 = 455$ cal./cm² 24 hrs. compared with the 530 incident calories. Since, however, with increasing latitude the atmospheric radiation preponderates, the heat balance is found to be zero with sufficient exactness.

The figures in Table 7 enable us to compare the greatest and smallest counter radiation of the atmosphere with the greatest and smallest daily solar radiation. It was to be expected that with increasing latitude the solar radiation at the time of the sun's greatest altitude would always exceed the counter radiation. It is surprising, however, to what extent the counter radiation of the atmosphere preponderates at the time when the sun is lowest. If we reduce the 181 calories of solar radiation which fall on latitude 50° by the value of the albedo, we obtain 110, while the counter radiation provides 460 calories. In central Europe the earth's surface receives two or three times as much heat from the counter radiation of the atmosphere as from solar radiation.

In order to be able to estimate the conserving effect of the atmosphere in another way, we have prepared Table 8.

TABLE 8.—*Showing the thermal conserving effect of the atmosphere. (Emden, p. 139, Table II.)*

North latitude.	Year.				Summer half-year.			Winter half-year.		
	Solar radiation.	Temperature.			Solar radiation.	Temperature.		Solar radiation.	Temperature.	
		Radiation.	Measured.	Difference.		Radiation.	July.		Radiation.	January.
	1	2	3	4	5	6	7	8	9	10
	Eq. da.	°C.	°C.	°C.	*Eq. da.*	°C.	°C.	*Eq. da.*	°C.	°C.
0°	365	26.5	25.9	−0.6	182.0	26.5	25.5	182.6	26.5	26.2
10°	360	25.6	26.3	0.7	193.3	37.8	27.9	166.9	19.1	23.9
20°	345	23.1	25.6	2.5	198.5	32.9	28.1	146.7	10.6	21.7
30°	321	17.1	20.3	3.2	198.4	32.9	27.4	122.6	− 2.9	13.9
40°	288.5	9.3	14.0	4.7	192.0	30.7	23.8	95.6	−18.2	3.9
50°	250	− 0.7	5.6	6.3	182.9	26.6	18.1	66.8	−40.0	− 7.2
60°	208	−12.8	− 0.1	12.7	169.5	21.1	14.1	38.2	−73.2	−16.0

We denote by "radiation temperature" the temperature to which the earth's surface is brought by the incident solar radiation on the assumption of gray radiation (temperature of radiation equilibrium). If the radiation does not last too short a time, say a few days, then the earth's surface would set itself at this temperature, since heat conduction plays only a secondary part. The second row contains these radiation temperatures for the different latitudes with average annual radiation. The observed mean annual temperatures of the atmosphere are only slightly higher, about 6°C. in latitude 50° north. This is an extraordinary quantity for climatic conditions, but from the thermodynamical standpoint, which here alone is important, we have to compare with the absolute temperature and we thus obtain a difference of only 2 per cent. *The average yearly heat conservation of the atmosphere is accordingly very small.* Columns 6 and 7 give the "radiation temperature" for the mean radiation of a summer half-year and the July temperatures. In the summer half-year the heat conservation of the atmosphere is negative.

The heat consevation in winter is surprisingly great. The mean winter radiation of the sun gives extraordinarily low radiation temperatures (they would be still lower for December radiation), even in north latitude 20° they lie 11 degrees below the January temperature. If there were no atmosphere, the 50th parallel would set itself at an average temperature of −40° in the course of the winter, and the 60th parallel would come to −73° C; the atmosphere raises these temperatures by 33 and 57 degrees, respectively. Consequently *in winter we obtain an extraordinarily large positive heat conservation, in summer a small negative one, and in the course of the year a small positive one.* These relations are repeated to a lesser degree during day and night.

Angot's well-known investigations on the distribution of the sun's radiation with reference to the atmospheric absorption are consequently of but small importance; they give only the influence of the atmosphere on the direct solar radiation. The smaller the transparency of the atmosphere, the more will the higher latitudes and the winter be set at a disadvantage in comparison with summer. If, however, we are dealing with the utility of the sun's radiation for the heating of the earth and its covering, the matter is quite different. The more the atmosphere emits (absorbs) the more does the sun's radiation pass in this roundabout way to the higher latitudes, to a large extent in winter but to a less extent in the whole year. In summer its heat conservation is negative, but in winter we benefit from solar radiation which is conveyed to the higher latitudes in a roundabout way by atmospheric radiation. The fact that the "radiation temperature" of central Europe is increased to the extent of 30 to 60 degrees (C.) by the counter radiation of the atmosphere shows that the atmosphere has not received its great capacity for radiating by being irradiated in these latitudes. The general circulation of the atmosphere—which is particularly active in winter on account of the large temperature differences—brings into higher latitudes masses of air capable of radiating and furnished with great quantities of entropy in the equatorial regions. The general circulation may be compared with a huge föhn, which ascends in the tropical belt of calm, flows over the trade-wind region and the horse latitudes and in descending is capable of radiating profusely on account of the high potential temperatures at moderate water vapor content.

In order to decide whether heat is conveyed to the higher latitudes and land masses chiefly only by solar radiation and atmospheric radiation, we have still to take into account the influence of the process of condensation. If water vapor condenses, quantities of entropy become available so that work and the yielding of heat can be combated. If the performance of work is sufficiently small, then in case the condensation takes place at 0°, the yield of heat is about 600 $\frac{\text{gram cal}}{\text{gram}}$ (at $-10°$, $0°$, $+10°$, it is 613, 607, and 589 cal., respectively). If we take a yearly rainfall of 120 cm, the condensation provides daily about 200 $\frac{\text{gram cal,}}{\text{cm}^2}$ while the counter radiation of the atmosphere is about three times this quantity. Now, Brückner[14] has shown that each drop of rain falls to the ground, on the average, three times before it is again returned to the ocean, consequently two-thirds of the heat made available by condensation is taken up from the land in evaporation. With a yearly rainfall of 120 cm, the quantity of heat brought by the water vapor taken from the ocean and made available by condensation is only 67 $\frac{\text{gram cal,}}{\text{cm}^2\text{, 24 hrs}}$ thus about 10 per cent of the counter radiation of the atmosphere, while over the ocean itself, the ratio is smaller. The gain of heat from condensing water vapor is small compared with the gain by radiation from radiating masses of air warmed in tropical regions.

The investigations of this paragraph were founded on a constant average content of water vapor in the atmosphere. The absorption capacity of the atmosphere as a function of the humidity is far too scantily known for us to be able to take into account with any certainty the variability of the humidity with regard to place and time. This quantity which is of such importance for the conservation of the earth's heat can be obtained by systematic measurements of the counter radiation of the atmosphere by means of the theory developed here.

[14] *Brückner, E.* On the origin of rain. Geographische Zeitschrift, 1900, 2:89.

A TORNADO IN UTAH.

By Arthur W. Stevens, United States Forest Service.

[The following report is transmitted through A. H. Thiessen, meteorologist, Salt Lake City, Utah, who remarks that the phenomenon is so very unusual in Utah that more than ordinary interest attaches to it.]

A small tornado occurred to-day (August 5, 1916) in the valley of the East Fork of the Sevier River about 1 mile north of Dave's Hollow ranger station, elevation 7,800 feet.

As observed from the station, the tornado appeared to have formed a short distance east of Frank Hatch's ranch on the Tropic Road. When first observed it was in the form of a slender inverted cone. It was too far distant to observe any whirling motion, but small puffs of cloud-like smoke were traveling rapidly upward on the outer surface of the cone.

The cone elongated rapidly and took on the shape of a rat's tail. The tip of the cloud did not touch the ground at any time, but directly under it was a large whirlwind that lifted a spray of mud and water from the ground.

The whirling cloud was white and at times appeared almost luminous, probably by contrast with the exceedingly dark cloud back of it.

The tornado lasted about 15 minutes, and traveled not over one-half mile in an easterly direction before breaking up.

After the storm I rode over the area of the storm, but as the only vegetation was sage brush and similar plants not over a foot high, there was no evidence to show exactly where the tornado had passed. The strip of ground affected was probably not over 20 or 25 feet wide, and possibly narrower than that.

This occurred during a period of exceptionally violent local rain and thunderstorms. Ranger Houston, of Dave's Hollow, pronounced it a waterspout, and I think it was so entered in his weather report for the day; but it was undoubtedly the same as the "twisters" of the Middle West, and the occurrence of one west of the Rockies was thought to be sufficiently rare to be worth recording.

THE GOVERNMENT SAFETY-FIRST TRAIN, 1916.

By Roy H. Finch, observer in charge of Weather Bureau Exhibit.

[Dated Weather Bureau, Washington, Sept. 25, 1916.]

During the summer of 1916 a unique method was pursued in informing the public as to some of the work that is being done by the various branches of the Federal Government.

While a great many people have had an opportunity to visit the different international expositions, and the National Capital, yet the vast majority of our citizens have but little knowledge of Federal activities.

After the Safety-First Convention, held in the New National Museum where most of the departments of the Government had exhibits, the idea was conceived of placing the exhibits on a railway train and giving everyone an opportunity to see what the Government is doing in the way of safeguarding life and property. Accordingly, a selection of exhibits was made and placed in 10 steel coaches, from which the seats had been removed, of the Baltimore & Ohio Railroad. Thus the Government Safety-First Train, a "World's Fair on wheels," originated, and on May 1, 1916, it started on its four months' educational trip.

The exhibits were arranged on both sides of the cars, or so as to leave a passageway from one end to the other,